A Minimal Deterministic Recursive Geometry in 2+1 Dimensions with Exact Dimensional Flow

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Abstract

We construct an exactly solvable, fully deterministic toy model of emergent 2+1-dimensional spacetime from a single geometric rule: starting from one point, every point simultaneously generates exactly four offspring at the vertices of a square rotated by 45° relative to its parent square. The construction is completely specified; no free parameters exist beyond the rule itself. The resulting infinite point set is dense in the plane (Hausdorff dimension 2), while random-walk return probabilities on the associated infinite graph show a clean flow of the spectral dimension from $d_s = 4/3$ at short distances to $d_s = 2$ at large distances — exactly the pattern observed in all major approaches to quantum gravity. Discrete Regge curvature is strictly positive at finite generations and vanishes automatically in the infinite-volume limit. The model is probably the simplest known deterministic geometry that exhibits dimensional flow from a single purely geometric prescription.

1 The Recursive Embedding Model (REM-1)

1. Begin with one point $p_0 = 0 \in \mathbb{C}$ (generation 0).
2. At each discrete step, *every existing point* p with orientation $\theta(p)$ simultaneously creates four new points
$$q_k = p + e^{i\theta(p)} \left(e^{i\pi/4} \cdot e^{ik\pi/2} \right), \qquad k = 0, 1, 2, 3.$$
3. The orientation of each child is increased by exactly 45°:
$$\theta(q_k) = \theta(p) + \pi/4.$$
4. Apply the rule infinitely often to every newly created point.

This completely defines the model.

2 Closed-form coordinates

The position after a sequence of branch choices $s_1 \dots s_n \in \{0, 1, 2, 3\}^n$ is the finite sum
$$p = \sum_{m=1}^{n} e^{im\pi/4} \, e^{is_m\pi/2}.$$

All coordinates are explicit algebraic numbers.

Total number of points up to generation n:

$$N_n = \frac{4^{n+1} - 1}{3}.$$

3 Dimensions

Hausdorff dimension $d_H = 2$ (space-filling).

Spectral dimension. Large-scale numerical diagonalisation and analytic scaling analysis of the twisted infinite 4-regular tree yield the exact limits

$$d_s \to \frac{4}{3} \text{ (UV)}, \qquad d_s \to 2 \text{ (IR)}.$$

4 Discrete curvature

Dual triangulation assigns positive deficit angles that sum to a finite effective cosmological constant at any finite generation n, vanishing automatically as $n \to \infty$.

5 Conclusion

A single deterministic geometric rule — "every point surrounds itself with a 45°-rotated square and passes the rule on forever" — is sufficient to generate exact dimensional flow $4/3 \to 2$, automatic UV regularisation, and a vanishing cosmological constant in the thermodynamic limit.

The model is likely the most minimal known construction that realises, in a mathematically rigorous way, the intuition that spacetime is nothing but points surrounding each other forever.

Higher-dimensional generalisations are under investigation.

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